Course Title: CALCULUS

Subject: Mathematics

Grade Level: 9-12

Duration: FULL YEAR

Prerequisite: PRE-CALCULUS (Honors or College Prep) WITH A GRADE OF B OR BETTER

Elective or Required: ELECTIVE

Mathematics Mission Statement

Mathematics is an integral part of our lives. Students must be actively involved in their mathematics education through the use of modeling and demonstrating the ability to persevere through problem solving. The mathematics curricula will emphasize critical thinking skills through a balance of logic and reasoning, attention to precision by utilizing patterns and structure, and bridging these ideas to cross-curricular learning. Students will be engaged and challenged in a student-centered learning environment that is developmentally appropriate and will communicate mathematical ideas, both in a verbal and written form. Through effectively applying hands-on manipulatives, basic computation skills and the use of technical writing to justify their processes, students will critique the work of themselves and others.

Course Description:

The Calculus course is designed for those students with a solid foundation in algebra, geometry, and math analysis. Students in this class should possess an interest in studying advanced mathematical topics as well as a desire to spend time solving problems that are of a challenging nature. The course is meant to serve as an introduction to derivatives and their applications as well as integration and its applications. The course covers a majority of the topics also covered in Advanced Placement Calculus, but not as quickly
or to the same depth. Therefore, this course is approximately equivalent to a one semester Calculus course at the college level.

The graphing calculator is used throughout the course to help students develop an intuitive feeling for concepts before they are approached through typical algebraic techniques. Although it is emphasized as a tool to illustrate ideas and topics, it is expected that students also become proficient in using the calculator to:

- Find roots of an equation
- Sketch functions in a specified window
- Approximate the derivative at a point using numerical methods
- Approximate the value of a definite integral using numerical methods.

The students are therefore required to supply their own graphing calculator and the teacher will very often use a graphing calculator view screen.

Author: Catherine McCarthy  
Date Submitted: Summer 2017  

Calculus

I Prerequisites for Calculus & Functional Analysis

Approximate # Of Weeks: 2

Essential Questions:
- What are the essential concepts one must understand about functions before beginning the study of calculus?
- How can functions be studied numerically, graphically, through tables, and analytically?
- What is the significance of a domain and range to function behavior?
- How can one recognize and represent a linear relationship?

NJ Student Learning Standards: A-CED#1, A-REI #3, F-IF #1 & 2, F-IF#8a, F-BF#1c, F-IF#7B, F-TF #.1, 2, &.5, 6, & 7

Upon completion of this unit students will be able to:
- Sketch graphs of equations by hand
- Find the x- and y-intercepts of graphs of equations
- Find the points of intersection of two graphs
- Use mathematical models to model and solve real-life problems
- Use the slope-intercept form of a linear equation to sketch graphs
- Find slopes of lines passing through two points
- Use the point-slope form to write equations of lines
- Find equations of parallel and perpendicular lines
- Use linear equations to model and solve real-life problems
- Decide whether the relationship between two variables is a function
- Find the domains and ranges of functions
- Use function notation and evaluate functions
- Combine functions to create other functions
- Find inverse functions algebraically

**Interdisciplinary Standards**

- Standard 9.1 21st Century Life & Career Skills
- Standard 8.1 Computer and Information Literacy
- Standard 8.2 Technology Education
- Standard 5.1 Science Practices

**Activities:**

- Assignments and announcements will be delivered through Google Classroom
- Smartboard Lessons will be used to relay notes to students
- Students will read and study material presented in course textbook and then be challenged with questions about their reading through examples.
- Students will take notes on instructor's lecture and participate in class discussions.
- Instructor will provide opportunity for both guided and independent practice.
- Homework assignments will be discussed to insure a good understanding of the prerequisites.
- Technology: the graphing calculator will be integrated into various exercises to help the student visualize the problems.
- Students will be asked to make conclusions after working through explorations scattered throughout the unit.
- Internet Activities: Interactive PreCalculus Review/Diagnostics
- Kahoot: Reviewing Functions
- Project: Number of Stores (sec. 1.2 p. 71)

**Enrichment Activities:**

- CalcView.com for worked-out solution videos
- LarsonAppliedCalculus.com for tutorial videos & Checkpoint worked-out solutions
- CalcChat.com for solutions to odd-numbered problems in text

**Methods of Assessments/Evaluation:**

- Written quizzes and unit test
- Worksheets
- Responses to discussion questions
- Homework
Classwork
- Verbal Assessment
- Think/Pair/Share
- Exit slips
- Written quizzes
- Self-Assessment exercises as found on textbook websites
- Independent extra credit assignments
- Observations

Resources/Including Online Resources
- Teacher Webpage
- Kahn Academy
- WebAssign for problem assignments

Learning Goals Scale:
4  Student will be able to apply a mathematical model to a given situation and solve a real-world problem
3  Student will be able to organize given information to form the equation of an appropriate function and sketch its graph
2  Student will be able to use function concepts in the solution to problems
1  Student will be able to identify characteristics, such as x- and y-intercepts, type of function, for a given graph

II LIMITS AND CONTINUITY

Approximate # Of Weeks: 3

Essential Questions:
- What is meant by a limit of a function?
- What are the graphical, numerical, and analytical ways to determine a limit of a function at a given value?
- How can a limit be used to determine the graphical behavior of a function?
- What is a continuous function?
- How does continuity depend on limits?

NJ Student Learning Standards:  F-IF.#6, 7b, & 8a, F-BF #1c, & F-LE.#5

Upon completion of this unit students will be able to:
- Define and describe a limit intuitively.
- Find limits using a numerical, graphical, and graphing calculator approach (using tables and graphs)
- Use different analytic techniques to evaluate limits of functions
- State and apply the properties of limits.
• Distinguish between and evaluate one-sided and two-sided limits
• Recognize unbounded behavior of functions.
• Define and describe continuity of a function at a point.
• Identify intervals over which a function is continuous.
• Identify types of discontinuities.
• User the greatest integer function to model and solve real-life problems
• Find vertical asymptotes of functions and find infinite limits
• Find the horizontal asymptotes of functions and find limits at infinity.
• Use asymptotes to answer questions about real-life situations.

Interdisciplinary Standards (njcccs.org)
• Standard 9.1 21st Century Life & Career Skills
• Standard 8.1 Computer and Information Literacy
• Standard 8.2 Technology Education
• Standard 5.1 Science Practices

Activities
• Students will watch Tutorials for the Calculus Phobe: Limits
• Smartboard Lessons will be used to relay notes to students
• Assignments and announcements will be delivered through Google Classroom
• Students will read and study material presented in course textbook and then be challenged with questions about their reading through examples.
• Students will take notes on instructor’s lecture and participate in class discussions.
• Instructor will provide opportunity for both guided and independent practice.
• Students will complete given classwork and homework assignment Homework assignments will be discussed to insure a good understanding of the prerequisites.
• Technology: the graphing calculator will be integrated into various exercises to help the student visualize the problems.
• Students will be asked to make conclusions after working through explorations scattered throughout the unit.
• Small group cooperative learning in reviewing homework
• Lab Activities: Investigating Limits Through Tables (Jim Rahn)
  Continuity Lab (Jim Rahn)
  Investigating Limits on the Calculator (Jim Rahn)
• Kahoot

Enrichment Activities:
• CalcView.com for worked-out solution videos
• LarsonAppliedCalculus.com for tutorial videos & Checkpoint worked-out solutions
• CalcChat.com for solutions to odd-numbered problems in text
• Kahn Academy: Limits Basics
  Limits from Equations
  Infinite Limits
  Continuity
Methods of Assessments/Evaluation:

- Written quizzes and unit test
- Worksheets
- Responses to discussion questions
- Homework
- Classwork
- Verbal Assessment
- Think/Pair/Share
- Exit slips
- Written quizzes
- Self-Assessment exercises as found on textbook websites
- Independent extra credit assignments
- Observations

Resources/Including Online Resources

- Teacher Webpage
- Handley Math Page Calculus Lesson Plans
  - [www.jamesrahn.com/homepages/calculus_labs.htm](http://www.jamesrahn.com/homepages/calculus_labs.htm)
  - [http://www.curvebank.calstatela.edu/limit/limt.htm](http://www.curvebank.calstatela.edu/limit/limt.htm)
- WebAssign for problem assignments
- Kahn Academy

Learning Goals Scale:

4  The student will be able to analyze a function graphically and analytically for its characteristics with respect limits and continuity over a specified domain
3  The student will be able to find the limit of a function given either a graph or an equation
2  The student will be able explain what it means to find a limit for a function
1  The student will be able to recognize where a function is continuous by looking at its graph

III DERIVATIVES

Approximate # Of Weeks: 12

Essential Questions:

- How can the limit of the average rate of change of a function lead to the instantaneous rate of change?
- How does one determine the slope of a function at a point?
- How does one find the derivative of a function?
- How are derivatives related to rates of change?
- What are the conditions for differentiability?
What are the various rules for differentiation and how are they applied?

**NJ State Learning Standards:** A-CED #1, A-REI #3, F-IF # 1, 2, 7b, 7c, 8a, F-BF #1c, F-LE #5

**Upon completion of this unit students will be able to:**
- Identify tangent lines to a graph at a point
- Approximate the slopes of tangent lines to graphs at points.
- Distinguish between average rate and instantaneous rate of change
- Use the limit definition to find the slopes of graphs at points.
- Use the limit definition to find the derivative.
- Use appropriate derivative notation.
- Approximate derivatives numerically using the calculator or graphically.
- Describe the 4 ways a derivative might fail to exist.
- Describe how differentiability implies continuity.
- Find the derivatives of functions using the Constant Rule.
- Find the derivatives of functions using the Power Rule.
- Find the derivatives of functions using the Constant Multiple Rule.
- Find the derivatives of functions using the Sum and Difference Rule.
- Use derivatives to answer questions about real-life situations.
- Find the average rates of change of functions over intervals.
- Find the instantaneous rates of change of functions at points.
- Find the marginal revenues, marginal costs, and marginal profits for products.
- Find the derivatives of functions using the Product Rule.
- Find the derivatives of functions using the Quotient Rule.
- Find the derivatives of functions using the Chain Rule.
- Find the derivatives of functions using the General Power Rule.
- Write derivatives in simplified form.
- Find higher order derivatives.
- Find and use a position function to determine the velocity and acceleration of a moving object.
- Find derivatives implicitly.
- Examine related variables and set up appropriate algebraic models.
- Solve related-rate problems.

**Interdisciplinary Standards**
- Standard 9.1 21st Century Life & Career Skills
- Standard 8.1 Computer and Information Literacy
- Standard 8.2 Technology Education
- Standard 5.1 Science Practices

**Activities:**
- Students will watch Tutorials for the Calculus Phobe: Derivatives
- Smartboard Lessons will be used to relay notes to students
• Assignments and announcements will be delivered through Google Classroom
• Students will read and study material presented in course textbook and then be challenged with questions about their reading through examples.
• Students will take notes on instructor’s lecture and participate in class discussions.
• Instructor will provide opportunity for both guided and independent practice.
• Students will complete given classwork and homework assignments. Homework assignments will be discussed to insure a good understanding of the prerequisites.
• Technology: the graphing calculator will be integrated into various exercises to help the student visualize the problems.
• Students will be asked to make conclusions after working through explorations scattered throughout the unit.
• Small group cooperative learning in reviewing homework
• Lab Activity: Derivatives by Jim Rahn
• Derivative Review Puzzle
• Kahoot
• Project: Cell Phone Subscribers in the US (P. 143)

Enrichment Activities:
• CalcView.com for worked-out solution videos
• LarsonAppliedCalculus.com for tutorial videos & Checkpoint worked-out solutions
• CalcChat.com for solutions to odd-numbered problems in text
• Kahn Academy: Derivative Introduction
  Basic Differentiation
  Product, Quotient and Chain Rule
  Differentiating Common Functions
  Advanced Differentiation

Methods of Assessments/Evaluation:
• Written quizzes and unit test
• Worksheets
• Responses to discussion questions
• Homework
• Classwork
• Verbal Assessment
• Think/Pair/Share
• Exit slips
• Written quizzes
• Self-Assessment exercises as found on textbook websites
• Independent extra credit assignments
• Observations

Resources/Including Online Resources
• Teacher Webpage
• www.jamesrahn.com/homepages/calculus_labs.htm
Learning Goals Scale:
4  The student will be able to analyze a real-life situation and use derivatives to find velocity, acceleration, marginal revenues, marginal costs, marginal profits, or solve a related rates problem
3  The student will be able to apply the appropriate method of differentiation for any algebraic function
2  The student will be able to use derivatives to find the slope of a graph at a point and the equation of the corresponding tangent line
1  The student will be able to recognize the difference between the average rate and instantaneous rate of change when solving problems

IV APPLICATIONS OF THE DERIVATIVE

Approximate # Of Weeks: 6

Essential Questions:
- How can derivatives be used to draw conclusions about extreme values of a function and the general shape of a function’s graph?
- How can optimizations problems be modeled algebraically using differentiation?
- How can a linearization model be used to find the zeroes of a function?

NJ State Learning Standards:  A-CED #1, A-REI #3, F-IF # 1, 2, 7b, 7c, 8a, F-BF #1c, F-LE #5

Upon completion of this unit students will be able to:
- Test for increasing and decreasing functions
- Find the critical values of functions and find the open intervals on which functions are increasing or decreasing
- Use increasing and decreasing functions to model and solve real-life problems.
- Recognize the occurrence of relative extrema of functions.
- Use the First-Derivative test to find the relative extrema of functions.
- Find absolute extrema of continuous functions on a closed interval.
- Find minimum and maximum values of real-life models and interpret the results in context.
- Determine the intervals on which the graphs of functions are concave upward or concave downward.
- Find the points of inflection of the graphs of functions.
- Use the Second-Derivative test to find the relative extrema of functions.
• Find the points of diminishing returns of input-output models.
• Solve real-life optimization problems.
• Recognize basic business terms and formulas.
• Solve business and economics optimization problems.
• Recognize the graphs of simple polynomial functions.
• Analyze and sketch the graphs of functions.
• Find the differentials of functions.
• Use differentials in economics to approximate changes in revenue, cost, and profit.
• Find the differential of a function using differentiation formulas.

**Interdisciplinary Standards:**
• Standard 9.1 21st Century Life & Career Skills
• Standard 8.1 Computer and Information Literacy
• Standard 8.2 Technology Education
• Standard 5.1 Science Practices

**Activities:**
• Smartboard Lessons will be used to relay notes to students
• Assignments and announcements will be delivered through Google Classroom
• Students will read and study material presented in course textbook and then be challenged with questions about their reading through examples.
• Students will take notes on instructor’s lecture and participate in class discussions.
• Instructor will provide opportunity for both guided and independent practice.
• Students will complete given classwork and homework assignment. Homework assignments will be discussed to insure a good understanding of the prerequisites.
• Technology: the graphing calculator will be integrated into various exercises to help the student visualize the problems.
• Students will be asked to make conclusions after working through explorations scattered throughout the unit.
• Small group cooperative learning in reviewing homework
• Class Activity: Derivative Matching Cards
• Lab Activity: Motion along a Line
  Maximization
  Understanding the Relationship between a Function and its Derivative
  Curve Sketching
  Newton’s Method all by Jim Rahn

**Enrichment Activities:**
• CalcView.com for worked-out solution videos
• LarsonAppliedCalculus.com for tutorial videos & Checkpoint worked-out solutions
• CalcChat.com for solutions to odd-numbered problems in text
• Kahn Academy: Analyzing Functions with Calculus
  Derivative Applications

Methods of Assessments/Evaluation:
• Written quizzes and unit test
• Worksheets
• Responses to discussion questions
• Homework
• Classwork
• Verbal Assessment
• Think/Pair/Share
• Exit slips
• Written quizzes
• Self-Assessment exercises as found on textbook websites
• Independent extra credit assignments
• Observations

Resources/Including Online Resources
• Teacher Webpage
• www.jamesrahn.com/homepages/calculus_labs.htm
• Handley Math Page Calculus Lesson Plans
• Kahn Academy
• http://www.univie.ac.at/future.media/moe/galerie/diff1/diff1.html
• http://www.themathpage.com/aCalc/max.htm#graph
• WebAssign for problem assignments

Learning Goals Scale:
4 The student will set up an appropriate model and apply the appropriate concepts of the derivative to solve real-life problems.
3 The student will, when analyzing the graph of a function, use a variety of problem solving strategies incorporating derivatives to investigate the important characteristics of the graph.
2 The student will summarize the characteristics of the graph of a function using the first and second derivative tests.
1 The student will calculate the first and second derivatives of a polynomial function.

V Exponential and Logarithmic Functions

Approximate # Of Weeks: 4

Essential Questions:
• How are the properties of exponential and logarithmic functions used to evaluate and simplify expressions?
How are exponential functions used to model and solve real-life situations?
How do we find the derivatives of exponential and logarithmic functions?
How do we use exponential growth and decay functions to model and solve real-life situations?

**NJ State Learning Standards:** A-CED #1, A-REI #3, F-IF # 1, 2, 7b, 7c, 8a, F-BF #1c, F-LE #5

**Upon completion of this unit students will be able to:**

- Use the properties of exponents to evaluate and simplify exponential expressions
- Sketch the graphs of exponential functions
- Evaluate and graph functions involving the natural exponential function
- Solve compound interest problems
- Solve present value problems
- Find the derivatives of natural exponential functions
- Use calculus to analyze the graphs of real-life functions that involve the natural exponential function
- Understand the definition of the natural logarithmic function, and sketch the graphs of natural logarithmic functions
- Use properties of logarithms to simplify, expand, and condense logarithmic expressions
- Use inverse properties of exponential and logarithmic functions to solve exponential and logarithmic equations
- Use properties of natural logarithms to answer questions about real-life situations
- Find the derivative of natural logarithmic functions
- Find the derivative of exponential and logarithmic functions involving other bases
- Use exponential growth and decay to model real-life situations

**Interdisciplinary Standards:**

- Standard 9.1 21st Century Life & Career Skills
- Standard 8.1 Computer and Information Literacy
- Standard 8.2 Technology Education
- Standard 5.1 Science Practices

**Activities:**

- Smartboard Lessons will be used to relay notes to students
- Assignments and announcements will be delivered through Google Classroom
- Students will read and study material presented in course textbook and then be challenged with questions about their reading through examples.
- Students will take notes on instructor’s lecture and participate in class discussions.
- Instructor will provide opportunity for both guided and independent practice.
- Students will complete given classwork and homework assignment Homewor assignment assignments will be discussed to insure a good understanding of the prerequisites.
- Technology: the graphing calculator will be integrated into various exercises to help the student visualize the problems.
- Students will be asked to make conclusions after working through explorations scattered throughout the unit.
- Small group cooperative learning in reviewing homework
- Project: ATM Surcharge Fee (P. 301)

**Enrichment Activities:**
- CalcView.com for worked-out solution videos
- LarsonAppliedCalculus.com for tutorial videos & Checkpoint worked-out solutions
- CalcChat.com for solutions to odd-numbered problems in text
- Kahn Academy: Composite Exponential Function Differentiation
  Differentiation of Logarithmic Functions

**Methods of Assessments/Evaluation:**
- Written quizzes and unit test
- Worksheets
- Responses to discussion questions
- Homework
- Classwork
- Verbal Assessment
- Think/Pair/Share
- Exit slips
- Written quizzes
- Self-Assessment exercises as found on textbook websites
- Independent extra credit assignments
- Observations

**Resources/Including Online Resources**
- Teacher Webpage
- Handley Math Page Calculus Lesson Plans
- WebAssign for problem assignments
- Kahn Academy

**Learning Goals Scale:**
4 The student will analyze a real-life situation and apply the properties of exponential and logarithmic functions to answer questions about questions about the problem.
3 The student will set up and solve a mathematical model for exponential growth and decay problems using calculus.
2 The student will apply the appropriate formulas in finding the derivatives of exponential and logarithmic functions.
1 The student will use the properties of exponents and logarithms to simplify expressions.
VI Integration and Its Applications

Approximate # of Weeks: 6

Essential Questions:
- How does instantaneous change accumulate over an interval to produce a function?
- How is an integral a representation of a summation?
- What is an indefinite integral?
- How can integration rules be used to find indefinite integrals?
- What is the difference between a definite and indefinite integral?
- How can the definite integral be applied to finding area under a curve?
- How does the limit of a Riemann Sum relate to integration?
- How does the Fundamental Theorem of Calculus connect integral and differential calculus?

NJ State Learning Standards: A-CED #1, A-REI #3, F-IF # 1, 2, 7b, 7c, 8a, F-BF #1c, F-LE #5

Upon completion of this unit students will be able to:
- Understand the definition of antiderivative and use indefinite integral notation for antiderivatives
- Use basic integration rules to find antiderivatives
- Use initial conditions to find particular solutions of indefinite integrals
- Use antiderivatives to solve real-life problems
- Use the General Power Rule to find indefinite integrals
- Use substitution to find indefinite integrals
- Use the General Power Rule to solve real-life problems
- Use the Exponentials Rule to find indefinite integrals
- Use the Log Rule to find indefinite integrals
- Understand the relationship between area and definite integrals
- Evaluate definite integrals using the Fundamental Theorem of Calculus
- Use definite integrals to solve marginal analysis problems
- Find the average values of functions over closed intervals
- Find the amounts of annuities
- Find the areas of regions bounded by two graphs
- Find consumer and producer surpluses
- Use the areas of regions bounded by two graphs to solve real-life problems
- Use the Midpoint Rule to approximate definite integrals
- Understand the definite integral as the limit of a sum

Interdisciplinary Standards:
- Standard 9.1 21st Century Life & Career Skills
- Standard 8.1 Computer and Information Literacy
- Standard 8.2 Technology Education
• Standard 5.1 Science Practices

Activities:
• Smartboard Lessons will be used to relay notes to students
• Assignments and announcements will be delivered through Google Classroom
• Students will read and study material presented in course textbook and then be challenged with questions about their reading through examples.
• Students will take notes on instructor's lecture and participate in class discussions.
• Instructor will provide opportunity for both guided and independent practice.
• Students will complete given classwork and homework assignments will be discussed to insure a good understanding of the prerequisites.
• Technology: the graphing calculator will be integrated into various exercises to help the student visualize the problems.
• Students will be asked to make conclusions after working through explorations scattered throughout the unit.
• Small group cooperative learning in reviewing homework
• Lab Activity: Investigating Riemann Sums Lab by Jim Rahn
  Understanding the 2nd Fundamental Theorem of Calculus by Jim Rahn

Enrichment Activities:
• CalcView.com for worked-out solution videos
• LarsonAppliedCalculus.com for tutorial videos & Checkpoint worked-out solutions
• CalcChat.com for solutions to odd-numbered problems in text
• Kahn Academy: Integration
  Integration Techniques
  Integration Applications

Methods of Assessments/Evaluation:
• Written quizzes and unit test
• Worksheets
• Responses to discussion questions
• Homework
• Classwork
• Verbal Assessment
• Think/Pair/Share
• Exit slips
• Written quizzes
• Self-Assessment exercises as found on textbook websites
• Independent extra credit assignments
• Observations

Resources/Including Online Resources
• Teacher Webpage
Learning Goals Scale:
4 The student will analyze a given problem so as to set up and solve it with an appropriate integral.
3 The student will use the concept of the antiderivative to evaluate integrals.
2 The student will use basic methods and the substitution method to find antiderivatives.
1 The student will when integrating, recognize that an indefinite integral denotes a family of antiderivatives, each differing by a constant \( C \), whereas a definite integral is a number.

SHOULD A LARGE PERCENTAGE OF STUDENTS BE INVOLVED WITH OTHER AP CLASSES, INCLUDE THE FOLLOWING UNIT DURING THE TWO WEEKS OF AP TESTING!

VII REVIEW FOR COLLEGE PLACEMENT TESTS

Approximate # of Weeks: 2

Essential Questions:
- What do I need to know when taking my college placement test?

Upon completion of this unit the student will be able to:
- Review those algebra, trigonometry and pre-calculus concepts necessary to successful complete a college placement test.

Activities:
- Students will work on their own college placement test practice questions as well as those which can be found online from other colleges such as Montclair University, Trinity College etc.

Methods of Assessment:
- Students will be quizzed on the concepts presented in the practice placement tests completed.

Supplemental Texts: